Timing Measurements for Nestor and NuBE

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Consider a particle traveling at the speed of light (β= 1) through water. This is applicable for the extremely high energy muons that Nestor and NuBE are interested in detecting. It emits Cerenkov photons at the angle $\Theta = \arccos(1/\beta n)$, where n is the refractive index of water. Since n = 4/3 this implies Θ = 41.4 degrees. Those Cerenkov photons then travel through the water at the speed of light in water, which is c/n = ¾ c. This situation is shown in Figure 1 where the particle trajectory crosses a horizontal plane containing a detector. The detector consists of 2 PMTs separated by distance d. The particle crosses the plane of the detector at a distance x from one PMT and at an angle ω to the vertical direction. Using basic geometry we can calculate the arrival time of the Cerenkov photons at the PMTs, and then the difference between those arrival times.

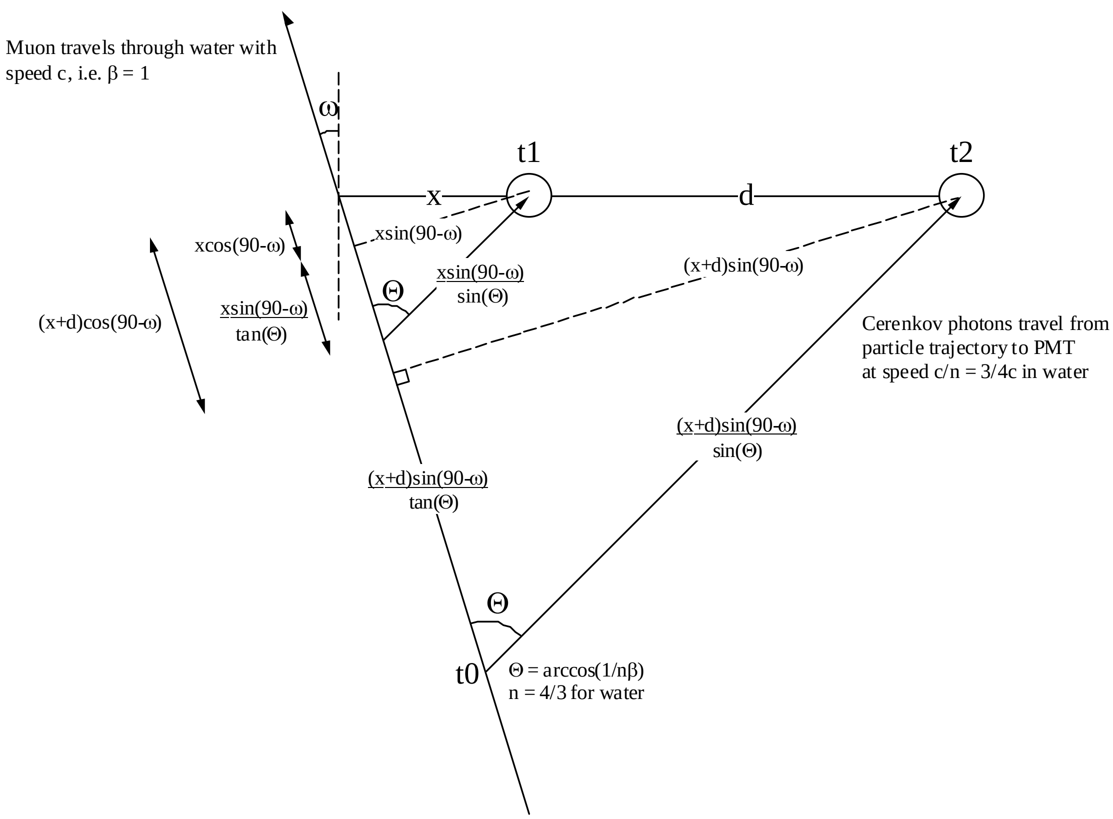

Figure 1: Diagram showing an upward going particle emitting Cerenkov photons towards 2 PMTs

After working through all the algebra the time difference between when the photons arrive at the PMTs is:

$$\Delta t = t2 - t1 = \frac{d}{c}\left(\tan(\Theta)\cos(\omega) - \sin(\omega)\right) \qquad (1)$$

It can be seen that x, the distance between where the particle crosses the plane of the detector and the nearest PMT, has dropped out. For a given PMT separation, d, and Cerenkov angle, Θ, the time difference between the PMT hits depends solely on the angle ω. Nestor and NuBE can use this relationship to reconstruct the angle ω of upward going muons by measuring Δt. This equation can be easily checked for a few geometrically simple cases, as shown in Table 1 Two values of d will be considered: 32m and 300m, which are respectively the diameter of a Nestor floor and the separation between two NuBE strings.

ω	Geometry	Δt / (ns)		
			d=32m	d=300m
0	Particle traveling vertically upwards	$\frac{d\tan(\Theta)}{c}$	94.1	881.9
$\frac{\pi}{2}$	Particle traveling horizontally from right to left	$\frac{-d}{c}$	-106.7	-1000.0
$\frac{-\pi}{2}$	Particle traveling horizontally from left to right	$\frac{d}{c}$	106.7	1000.0
Θ	Particle traveling at the Cerenkov angle to the vertical so that the Cerenkov photons themselves are traveling vertically	0	0	0
-(90-Θ)	Particle traveling at the angle such that the Cerenkov photons are traveling horizontally from left to right	$\frac{d}{c\cos(\Theta)}$	142.2	1333.3

Table 1: Time Difference for selected values of the angle ω

In Figure 2 the solid line shows how the dimensionless quantity $\left(\tan(\Theta)\cos(\omega) - \sin(\omega)\right)$ varies with the crossing angle ω for all upward going particles.

Both the Nestor and NuBE experiments will use measurements of time differences to reconstruct the angle of the muon velocity vector. The error on the time difference measurement, σ(Δt), will then lead to an error in the reconstructed angle, σ(ω):

$$\sigma(\omega) = \frac{\partial(\omega)}{\partial(\Delta t)}\sigma(\Delta t) \qquad (2)$$

In order to calculate the worst value of σ(Δt) that will yield the required value for σ(ω) it is necessary to invert this equation:

$$\sigma(\Delta t) = \frac{\partial(\Delta t)}{\partial(\omega)}\sigma(\omega) \qquad (3)$$

We can evaluate this expression by differentiating equation 1:

$$\sigma(\Delta t) = \frac{-d\sigma(\omega)}{c}\left(\tan(\Theta)\sin(\omega) + \cos(\omega)\right) \qquad (4)$$

$\sigma(\Delta t)$ is in units of radians/second. The dashed line in Figure 2 shows how the dimensionless quantity $-\left(\tan(\Theta)\sin(\omega) + \cos(\omega)\right)$ varies with the crossing angle ω. Since this quantity is not a constant it implies that if an experiment measures time differences with a fixed resolution, σ(Δt), the resulting angular resolution, σ(ω), will vary as a function of the angle ω. The angular resolution requirements for Nestor and NuBE are specified for vertically going muons, i.e. ω = 0 degrees. In this situation equation (4) reduces to:

$$\sigma(\Delta t) = \frac{-d\sigma(\omega)}{c} \times \left(\frac{\pi}{180}\right) \qquad (5)$$

The factor of (π/180) is introduced to convert from radians/second to degrees/second. The requirements and the necessary values of σ(Δt) are shown in Table 2:

d / (m)	Δt / (ns)	Required value of σ(ω) / (degrees)	σ(Δt) / (ns)	σ(t) / (ns)
Nestor: 32	94.1	1.0	1.86	1.3
NuBE: 300	881.9	1.0	17.4	12.3
NuBE: 10	29.4	30.0	17.4	12.3

Table 2: σ(Δt) and σ(t) for Nestor and NuBE

It can be seen that there are 2 requirements for NuBE. A small angular resolution is required for high energy, long range, muons whose photons are detected by multiple strings. However, for calibration and monitoring purposes, it is required that NuBE be able to reconstruct the direction of a lower energy muon from hits on a single node. A larger angular resolution is acceptable here.

The quantities that the experiments actually measure are the arrival times of photons at the PMTs, i.e. t1 and t2, not Δt. Assuming that σ(t1) and σ(t2) are equal in magnitude and uncorrelated, then $\sigma(t) = \sigma(\Delta t)/\sqrt{2}$. The values of σ(t) are shown in the final column of Table 2.

In order for Nestor to meet its angular resolution requirements it would need to measure the arrival time of photons at the PMTs with an error of no more than 1.3 ns. This is almost the size of the error due to the transit time spread of the PMTs themselves. Since the PMTs have become the limiting factor the requirement for the Nestor electronics is that its contribution to the final resolution be less than that of the PMTs. Since Nestor actually contains many more than just 2 PMTs it should still be possible to meet their required angular resolution.

For NuBE the situation is different; the required resolution does not yet approach the limit set by the PMTs. So long as the electronics can record the arrival time of photons with an accuracy of $\sigma(t)$ = 10 ns NuBE will be able to achieve its angular resolution.

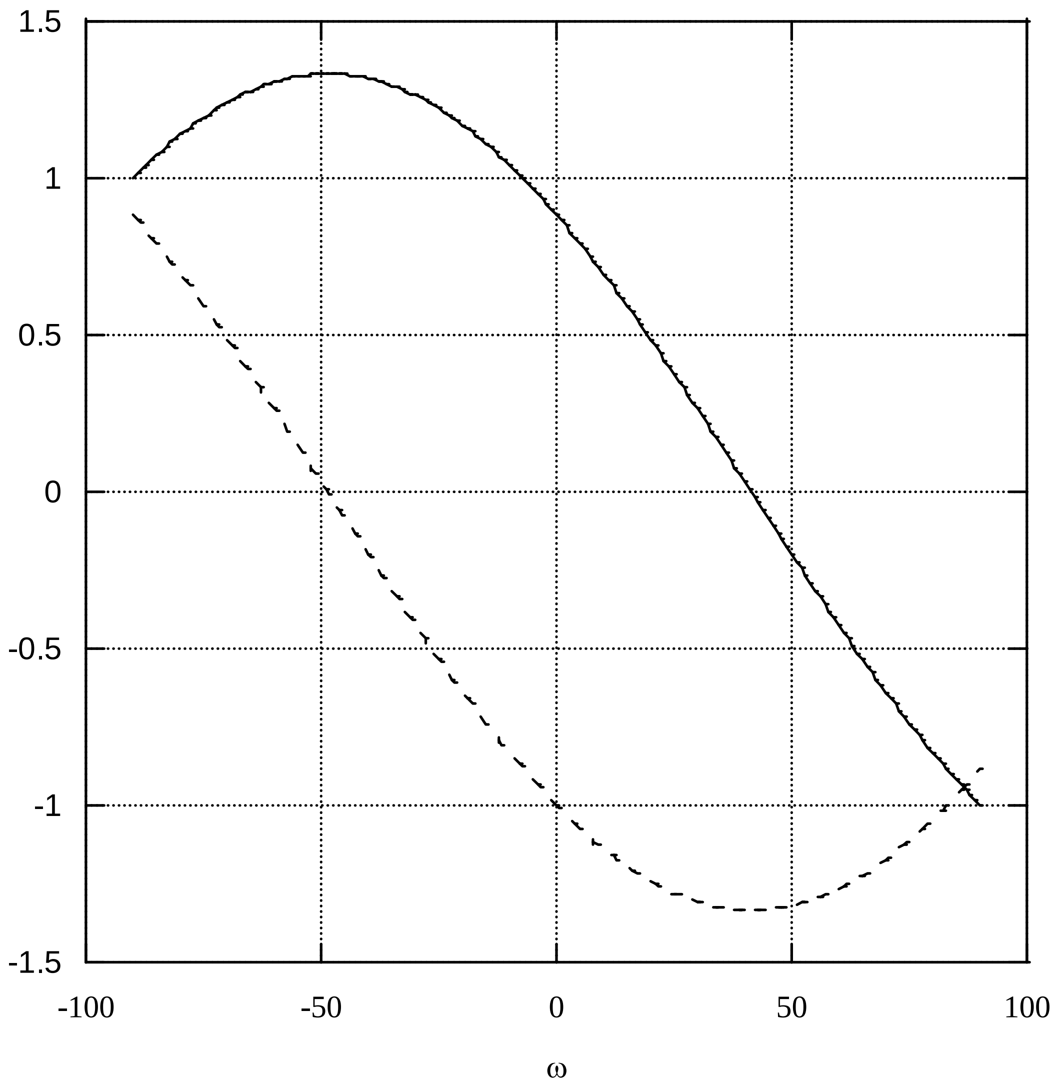

Figure 2: The dimensionless quantities $\left(\tan(\Theta)\cos(\omega)-\sin(\omega)\right)$ (solid line) and $-\left(\tan(\Theta)\sin(\omega)+\cos(\omega)\right)$ (dashed line) as a function of ω